ALLOWANCE FOR THE EFFECT OF PRESSURE IN THE THEORY OF SECOND-ORDER PHASE TRANSITIONS (WITH APPLICATION TO THE CASE OF SUPERCONDUCTIVITY)

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Submitted to JETP editor January 26, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 2104-2106 (June, 1963)

The effect of pressure on second-order phase transitions is discussed. Specifically, the superconducting transition is considered.

In the theory of second-order phase transitions one usually considers only the temperature dependence of a number of quantities near the transition point.[1] Nevertheless the pressure dependence, for example the dependence of the characteristic transition parameters η_i, is also of interest. The components of the spontaneous electrical polarization or magnetization vectors are usually taken as η_i in ferromagnetics and ferroelectrics. In superconductors $\eta^2 \sim |\Psi|^2$, where Ψ is the effective wave function of superconducting electrons,[2] and in pure isotropic superconductors $\Psi \sim \Delta$, where Δ is the gap in the energy spectrum.[3] Within the framework of the approximate theory of phase transitions[1] allowance for pressure is quite obvious. Nevertheless we shall consider here this problem with reference to superconductors for the following reasons. In the region of the superconducting transition at $T \neq 0$ the theory[1,2] is practically exact.[4] Moreover, in the case of superconductivity, using the microscopic BCS theory,[5] it is possible to obtain some additional information for the model considered. Finally, it is in the case of superconductors that the pressure dependence of various quantities can be determined easily by experiment and in particular, as follows from the work of Brandt and N. I. Ginzburg,[6] one can hope to study in sufficient detail the dependence $T_c(p_c)$.

Bearing in mind the application to superconductivity we shall from the outset denote the characteristic parameter η by Δ and we shall write the well-known expansion of the thermodynamic potential in the form (the quantity Δ is assumed to be real, otherwise Δ^2 should be replaced by $|\Delta|^2$)

$$\Phi(p, T) = \Phi_0(p, T) + \alpha(p, T)\Delta^2 + \tfrac{1}{2}\beta(p, T)\Delta^4 + \ldots \quad (1)$$

Along the line of the second-order phase transition $\alpha(p_c, T_c) = 0$. At a fixed pressure near the transition point we can assume

$$\alpha(p_c, T) = \left(\frac{\partial\alpha}{\partial T}\right)_{p_c, T_c}(T - T_c),$$

as is usual.[1-4] Similarly at a fixed temperature we can assume

$$\alpha(p, T_c) = (\partial\alpha/\partial p)_{p_c, T_c}(p - p_c).$$

Moreover, at equilibrium in the ordered phase $\Delta^2 = -\alpha/\beta$. Hence we obtain the dependence

$$\Delta(p, T_c(p_c)) = \left[\frac{(\partial\alpha/\partial p)_{p_c, T_\kappa}}{\beta(p_c, T_c)}(p_c - p)\right]^{1/2}. \quad (2)$$

Here of course $T_c(p_c)$ denotes the temperature at which measurements are carried out at the pressure p (the temperature $T_c(p_c)$ is the critical temperature only for the pressure p_c). The dependence $\Delta(p)$ can be measured by various methods. At the same time it follows from the expansion (1) that $H_{cb}^2/8\pi = \alpha^2/2\beta$ (cf.[2]), i.e., the critical magnetic field for bulk samples $H_{cb}(p, T_c(p_c)) \propto (p_c - p)$.

For the λ-transition in liquid helium ρ_s (cf.[7]) is taken as Δ^2 in Eq. (1), and therefore $\rho_s(p, T_\lambda(p_\lambda)) \propto (p_\lambda - p)$, if one ignores the possible need of allowing in Eq. (1) for a term of higher order [i.e., a term $(1/6)\gamma\rho_s^3$]. However, in this case the region of applicability of the expansion of type (1) is not sufficiently clear. This difficulty makes a study of the dependence $\rho_s(p, T)$ near the λ-point even more interesting.

The shape of the curve $p_c(T_c)$ for which $\alpha(p_c, T_c) = 0$ is in general not known. Moreover, for second-order transitions we cannot even state that necessarily $dp_c/dT_c \to 0$ as $T \to 0$. In fact, for a second-order transition